# *The Hitchhiker's Guide to Harmony Interactions*

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**Abstract**

*In this poster, I use harmony systems with multiple feature spreadings as a litmus test for the possible configurations of items involved in different harmonies. I investigate relations between harmonic sets within the same language, and show that the only unattested case is when the harmonic sets are incomparable, i.e. share a non-empty intersection while not being in the set-subset relation. Surprising at the first glance, I then show that eliminating such option helps to remove the vast majority of guesses from the set of initial hypotheses.*

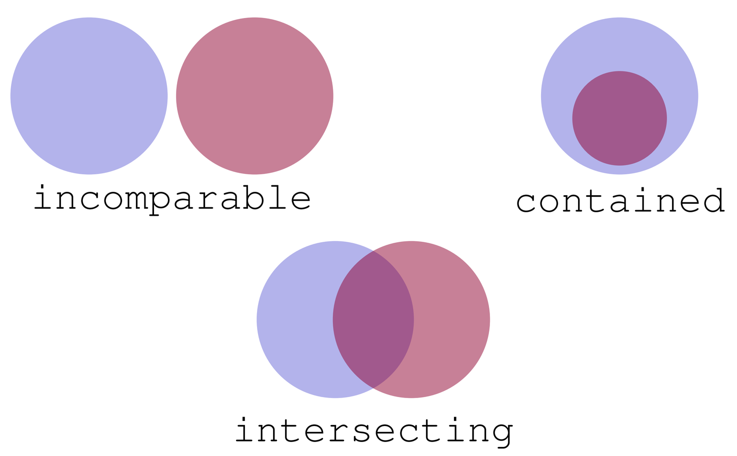
## Introduction

**What is the main question?**
When there is more than one harmony within the same language, what are the possible relations among the sets of items involved in either of the harmonies?

**Why should we care?**
This is about the importance of eliminating possibilities that do not appear in natural language, understanding its properties better, and bringing naturalness in the formal descriptions of natural language patterns.

For instance, as it was highlighted by [5, 8]: for a domain with $n$ elements, there are $2^{4^n}$ possible generalized quantifiers. However, when we take into account such property of natural language quantifiers as conservativity, it reduces the number of options to $2^{3^n}$. For example, for a domain with $2$ elements, there are $65536$ possible generalized quantifiers, but only $64$ of them are conservative.

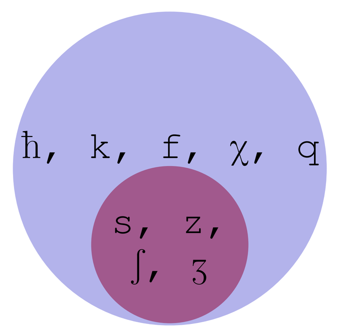
**What is the result?**
Theoretically possible relations between two sets of harmonizing elements are containment ({a,b,c} and {a,b}), disjunction ({a,b} and {c,d}), and intersection ({a,b} and {b,c}).




I show that the latter case in unattested. This restriction actually reduces the amount of harmonic sets configurations. For a set of $10$ elements, there are $511$ ways to form two disjoint sets, $1022$ ways to arrange them with respect to the containment relation, and $27990$ ways to form two sets with a non-empty intersection. The difference is striking: in this case, by removing the intersection relations, the amount of possible harmonic sets arrangements will be reduced by $95\%$.

## 1 Contained sets

The first possibility is to have two sets of harmonizing elements, where one set is the subset of the other one. Example of the language that involves this type of dependency is IMDLAWN TASHLHIYT that is discussed below.




**Harmonies in IMDLAWN TASHLHIYT (Berber)**

In IMDLAWN TASHLHIYT, affixal sibilants regressively harmonize with the stem in voicing and anteriority, see [4, 7][a]. Whereas the anteriority harmony is not a subject for blockers of any kind, the voicing assimilation is blocked by any intervening voiceless obstruents. If there are no sibilants in the stem, the underspecified affixal element is realized as the voiceless anterior sibilant [s]. The data in (13-22) from [2, 3] illustrate the harmonic pattern using the causative prefix *s*-.

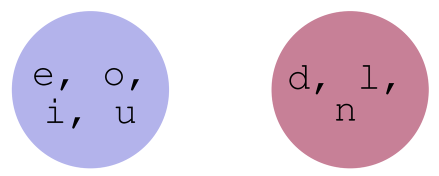
(1) **s**-a**s:**twa 'CAUS-settle'
(2) **z**-bru**z:**a 'CAUS-crumble'
(3) **s:**-uk**z** 'CAUS-recognize'
(4) **ʃ**-fia**ʃ**r 'CAUS-be.full.of.straw'
(5) **ʒ**-m:**ʒ**dawl 'CAUS-stumble'
(6) **ʃ**-qu**ʒ:**i 'CAUS-be.dislocated'

[a] The generalization presented here is simplified. Refer to [7] for the details.

For a set of $n$ elements, the amount of its subsets is equal to choosing $k$ elements from a set of $n$, or $\binom{n}{k}$. The amount of all other proper subsets is given by $\sum_{k=1}^{n-1}\binom{n}{k} = 2^n - 2$.

## 2 Incomparable sets

Another possibility for the two sets of harmonizing elements is to have no elements in their intersection. For an example of such a configuration, see the harmonies in KIKONGO below.




**Harmonies in KIKONGO (Bantu)**

In this language, suffixes are specified for rounding, and acquire their height specification depending on the stem vowel.

(7) -somp-**e**l- 'attach-APPL'
(8) -tomb-**o**l- 'do-TRANS'
(9) -sik-**i**l- 'support-APPL'
(10) -vil-**u**l- 'move-TRANS'

But along with vowel harmony, this language also has nasal agreement. Affixal /d/ and /l/ become /n/ if nasal consonants such as /m/ or /n/ are found in the root, [1].

(11) -suk-**idi**- 'wash-PERF.ACT'
(12) -**n**ik-**ini**- 'ground-PERF.ACT'
(13) -suk-**ulu**- 'wash-PERF.PASS'
(14) -**n**ik-**unu**- 'ground-PERF.PASS'

The general case of partitioning a set of $n$ elements into $k$ disjoint subsets is given by Stirling Numbers of the Second Kind, see [6].

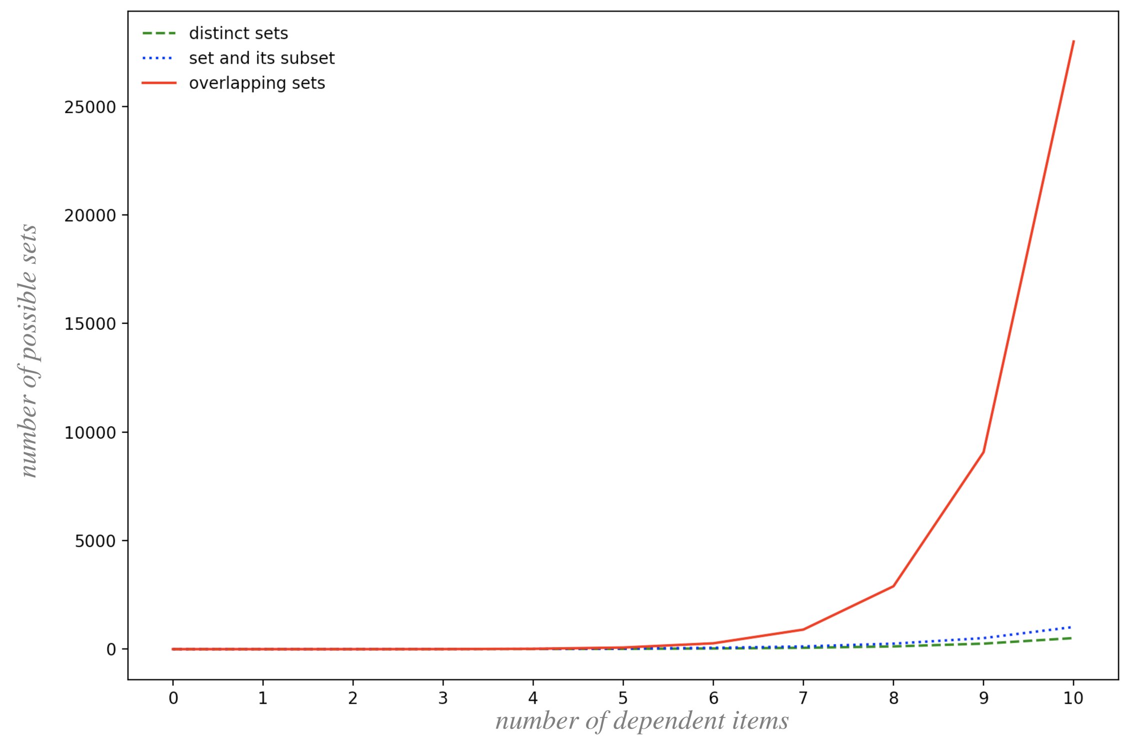
For the partitioning into $2$ sets, the following formula can be used: $S(n,2) = \frac{1}{2}\sum_{j=0}^{2}(-1)^{2-j}\binom{2}{j}j^n$.

## 3 Intersecting sets

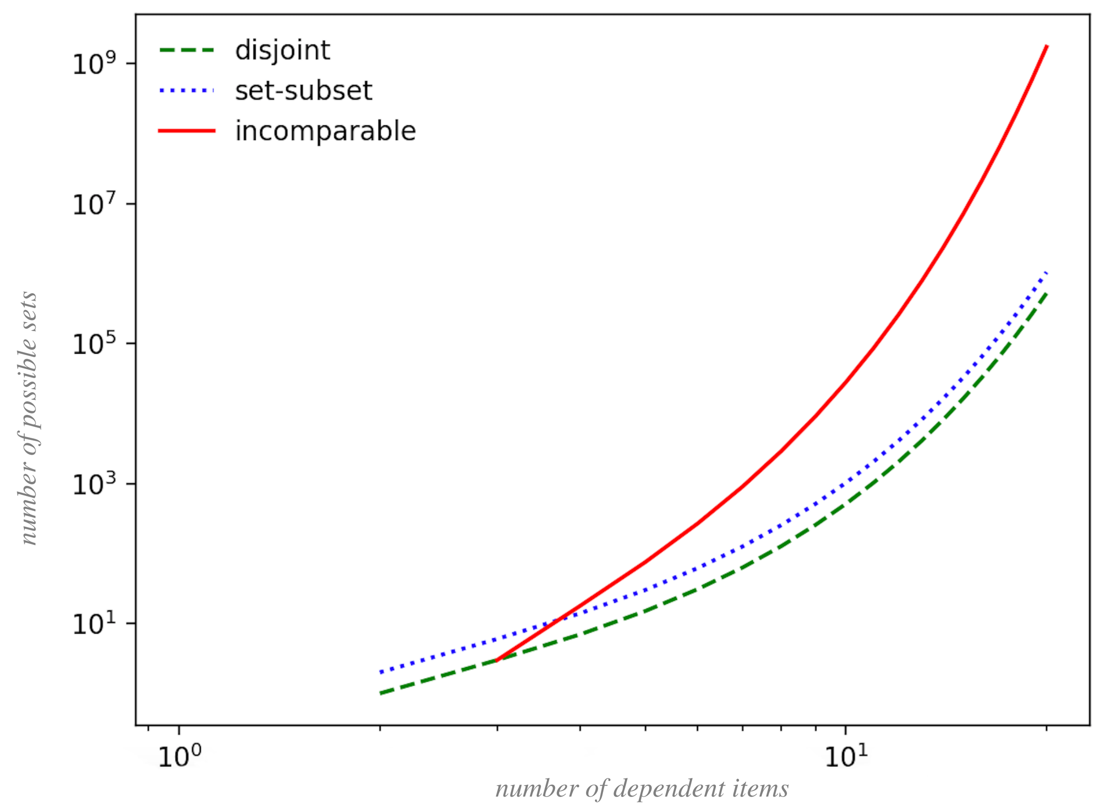
There are no attested language where two harmonies operate over only partially overlapping sets of elements. The problem of calculating the number of such sets can be divided into two sub-problems: partitioning the set of $n$ elements into $3$ disjoint sets using $S(n,3)$; and ordering the partitions to generate all possible intersections. The solution is the following formula: $3 * S(n,3) = \frac{1}{2}\sum_{j=0}^{3}(-1)^{3-j}\binom{3}{j}j^n$.

## Conclusion

The pictures below illustrate growth of these functions. For example, for $10$ elements, there are $511$ ways to arrange them in two distinct sets, $1022$ ways to obtain a set and its subset, and $27990$ ways to create two overlapping sets.

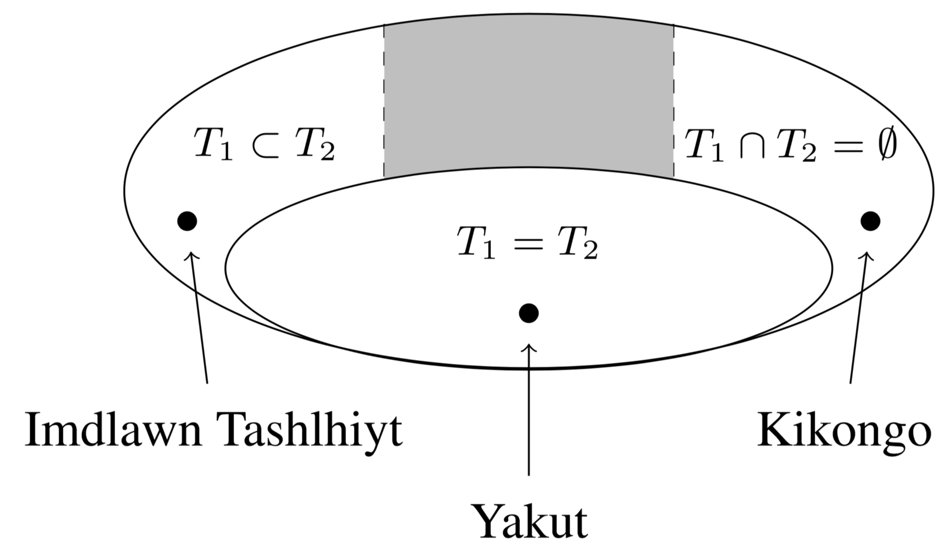



In a log-log representation, the growth of the function can be depicted as follows:




This is just preliminary research about the typology of long-distance processes and the math behind it, and, of course, a lot remains to be explored. However, this result is interesting from at least the following perspectives:

- it reveals new typological generalizations about harmonic systems and natural languages in general;
- it brings a desired naturalness to the models of natural languages.

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