Introduction

What is the main question?

When there is more than one harmony within the same language, what are the possible relations among the sets of items involved in either of the harmonies?

Why should we care?

This is about the importance of eliminating possibilities that do not appear in natural language, understanding its properties better, and bringing naturalness in the formal descriptions of natural language patterns. For instance, as it was highlighted by [5, 8], for a domain with n elements, there are $2^n$ possible generalized quantifiers. However, when we take into account the property of natural language quantifiers as conservativity, it reduces the number of options to $2^n$. For example, for a domain with 2 elements, there are 65,536 possible generalized quantifiers, but only 64 of them are conservative.

What is the result?

Theoretically possible relations between two sets of harmonizing elements are containment (a,b,c) and (a|b), disjunction ((a,b) and (c,d)), and intersection ((a|b) and (b,c)).

I show that the latter case in untested. This restriction actually reduces the amount of harmonic sets configurations. For a set of 10 elements, there are 511 ways to form two disjoint sets, 1022 ways to arrange them with respect to the containment relation, and 2799 ways to form two sets with a non-empty intersection. The difference is striking: in this case, by removing the intersection relations, the amount of its subsets is equal to choosing $k$ elements from a set of $n$, or $\binom{n}{k}$. The amount of all other proper subsets is given by $\sum_{j=0}^{n-1} \binom{n}{j} = 2^n - 2$.

1 Contained sets

The first possibility is to have two sets of harmonizing elements, where one set is the subset of the other one. Example of the language that involves this type of dependency is IMDLAWN TASHILHIYI that is discussed below.

For the partitioning into 2 sets, the following formula can be used:

$$S(n, 2) = \sum_{j=0}^{n-1} \binom{n}{j} = 2^n - 2$$

3 Intersecting sets

There are no attested language where two harmonies operate over only partially overlapping sets of elements. The problem of calculating the number of such sets can be divided into two sub-problems: partitioning the set of n elements into 1 disjoint sets using $S(n, 1)$, and ordering the partitions to generate all possible intersections. The solution is the following formula:

$$3 \cdot S(n, 3) = \sum_{j=0}^{n-1} \binom{n}{j} - 1$$

Conclusion

The pictures below illustrate growth of these functions. For example, for 10 elements, there are 511 ways to arrange them in two disjoint sets, 1022 ways to obtain a set and its subset, and 2799 ways to create two overlapping sets.

References


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